Investigation of the Dynamic Stability for a Light Aircraft

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To cite this article:

Received: November 22, 2018; Accepted: December 11, 2018; Published: January 15, 2019

Abstract: The dynamic stability of a light aircraft is very crucial at all phases of flight. This may include takeoff, climb, cruise, loiter, descend and landing where the aircraft is subjected to intense pressure from aerodynamic forces and moments. Control surfaces and flight control systems are therefore, used to control and pilot the aircraft to safe flight. The dynamic behavior of the aircraft can be simulated if an appropriate model of the aircraft is generated with a view to predicting the amount of force required to control the actuators that would actuate the control surfaces and make the aircraft stable from a disturbance. In this research paper, the dynamic stability of a light aircraft called the Air Beetle (ABT-18) was investigated where the geometry of the aircraft was inputted in Athena Vortex Lattice (AVL) Software using X downstream, Y outright wing and Z up coordinates. The objective was to investigate how stable the aircraft will be on the longitudinal and lateral directions respectively. A model of the aircraft was created with dimensionless aerodynamic coefficients based on trim flight condition of cruise speed 51.4m/s at 12,000ft altitude. The aircraft airframe configuration and specification was inputted in AVL and aerodynamic stability coefficients were produced. The simulation was carried out in the graphic environment of Matlab Simulink, where block models of the aircraft were formed. Thereafter, transfer functions were obtained from the solutions of the light aircraft equations of motions. Pole placement method was used to test the dynamic stability of the aircraft and it was found to be laterally stable on the longitudinal axis and longitudinally stable on the lateral axis. Thus, the dynamic stability controls of the aircraft were achieved in autopilot design by implementing PID controllers’ successive loops and it was found that the ABT-18 aircraft had satisfied the conditions necessary for longitudinal and lateral stabilities.

Keywords: Air Beetle (ABT-18), Athena Vortex Lattice (AVL), Proportional, Integrator, Derivative, (PID) Controllers, Lateral, Longitudinal, Directional, Dynamic Stabilities

1. Introduction

The autopilot system is integrated in the aircraft as one of the systems to reduce the pilot workload, improve the dynamic stability of the aircraft as it performs its mission. This autopilot operates on longitudinal, lateral or directional axis and possesses autopilot modes such as the altitude hold, heading hold, roll angle hold, pitch angle hold, coordinated turn and it is embedded into the flight management system for trajectory, guidance and path following. Therefore, the dynamic stability for a light aircraft was investigated on the longitudinal and lateral directional axes as the main autopilot operating axes based on autopilot modes such as the altitude hold, heading hold, roll angle hold, pitch angle hold, coordinated turn with FMS guidance for trajectory and path following. The results showed that the aircraft is laterally and longitudinally stable.

This increase usage of aircraft by the military and civil operators for reconnaissance, surveillance and other missions motivated the researchers to embark on a study, to investigate the dynamic responses or stabilities of a light aircraft, the Air Beetle, ABT-18 with a view to assigning the results of our findings in a project to convert a light aircraft to an Unmanned Aerial Vehicle (UAV). The project and results of our findings can be applied to similar aircraft of same category used for the purpose of reconnaissance and surveillance activities like pipelines vandalisation, border patrols in the area of security, as well as, agricultural works. The light aircraft used is a two-seater, low wing, primary trainer aircraft that provides its pilots and trainee pilots with initial flight experience. It is designed to be capable of aerobatic flight and has been earmarked for conversion into an UAV at a later period, hence the need to investigate the
dynamic behaviour of its structure and stability at different flight profiles.

The conceptual design of the light aircraft was carried out by the staff of the aircraft engineering department of Air force Institute of Technology Kaduna and it forms the basis for the design specifications, of a prototype UAV expected to be robust, lighter, safer, reliable, and easily maintained at a reduced cost with the following preliminary design specifications; [1]

<table>
<thead>
<tr>
<th>Table 1. Preliminary Design Specification of a Light Aircraft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty weight</td>
</tr>
<tr>
<td>Maximum take-off weight</td>
</tr>
<tr>
<td>Cruise speed</td>
</tr>
<tr>
<td>Landing gear (Undercarriage)</td>
</tr>
<tr>
<td>Ceiling</td>
</tr>
<tr>
<td>Endurance</td>
</tr>
<tr>
<td>Service life</td>
</tr>
</tbody>
</table>

Some basic assumptions were considered in the course of the research like the gain of the lifting surfaces of the aircraft and the feedback systems such as rate gyro, AHRS etc. were taken to be unity. The lift produced by the fuselage of the aircraft was negligible as the AVL software analyses better without fuselage and lesser number of sections. Finally, the autopilot system to be modelled, had a disturbance rejection ratio.

It was observed that the AVL Software used had no library for the NACA 5 or NACA 6 series airfoils and as such, could not import the NACA 23013:5 and NACA M3 airfoils of the light aircraft, hence, similar airfoils of these kinds or types were used in order to make analysis on AVL. Again, the information needed for the successful running and analysis of the AVL Software was not readily available and the MATLAB Simulink did not have a platform in place to read the output of the AVL so as to enable the researchers create a model for the light aircraft dynamics for MATLAB simulations. These difficulties pushed us into researching and comparing the data with DATCOM Software.

The AVL, however, generated aircraft stability derivatives using the light aircraft lifting surfaces estimated based on the steady vortex shedding of the surfaces at small angles of attack and sideslip. In AVL model, the fuselage effects were ignored. Hence, each major airfoil section was defined. For instance, the aircraft wing which generates the highest amount of lift, had the center airfoil section, where the flap or aileron starts and ends. The airfoil sections where the sweep angle and the dihedral angle change respectively, were all defined so that the geometry will exactly match the actual aircraft. These also allows for the creation of v-tail geometry models directly, which was graphically represented in three dimensions, allowing for increased troubleshooting ability. The stability derivatives about the center of gravity were then calculated using the lifting surfaces geometry, while the angles of attack of the aircraft were varied based on intervals of one unit setting in our investigations. The stability derivatives were determined for each angular position and these were outputted to an.XML file or.dat file. [8]

2. Equations for Lateral and Longitudinal Dynamics

The dynamics of the ABT-18 light aircraft to be modelled as UAV was decoupled into lateral dynamics and longitudinal dynamics. The lateral dynamics are the aircraft’s response along the roll and yaw axes, since the lateral modes are generally excited with aileron and rudder inputs. The longitudinal dynamics are the response of the aircraft along the pitch axis and is controlled by the elevator, hence, two space equations were generated for both the longitudinal and lateral dynamics motions respectively.

2.1. The Equation of Motion for the Longitudinal Dynamics Was Generated As

\[
\begin{bmatrix}
\dot{\psi} \\
\dot{q} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
x_u & x_w & x_q & x_0 \\
0 & m_u & 0 & 0 \\
0 & 0 & m_w & 0
\end{bmatrix}
\begin{bmatrix}
u \\
w \\
q \\
\dot{\theta}
\end{bmatrix} +
\begin{bmatrix}
x_n \\
0 \\
0 \\
0
\end{bmatrix} [\eta] (1)
\]

State Space Equation of Longitudinal Motion

Here, \(x_u = \) Concise axial force due to velocity, \(x_w = \) Concise axial force due to “incidence” \(x_q = \) Concise axial force due to pitch rate, \(z_u = \) Concise normal force due to velocity, \(z_w = \) Concise normal force due to “incidence”, \(z_q = \) Concise normal force due to pitch angle, \(m_u = \) Concise pitching moment due to velocity, \(m_w = \) Concise pitching moment due to “incidence”, \(m_q = \) Concise pitching moment due to pitch rate, \(m_\theta = \) Concise pitching moment due to pitch angle, q \(= \) pitch rate, \(z_\theta = \) Elevator angle perturbation, \(x_\eta = \) Concise axial force due to elevator, \(z_\eta = \) Concise normal force due to elevator, \(m_\eta = \) Concise pitching moment due to elevator.

A Solution of the Equation for the Longitudinal Dynamics Was Summarized as

\[
y(t) = f(t) \begin{bmatrix} 1 & 0 & 0 & 0 \ 0 & 1 & 0 & 0 \ 0 & 0 & 1 & 0 \ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\
w \\
q \\
\theta
\end{bmatrix} (2)
\]

Solution of the State Space equation of longitudinal dynamics of ABT 18 light Aircraft
2.2. The Equation of Motion for the Lateral Directional Dynamics Axis Was Given as

\[
\begin{bmatrix}
\dot{\nu} \\
\dot{\rho} \\
\dot{\gamma} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
y_p & y_p & y_r & y_p \\
\dot{\rho}_p & \dot{\rho}_p & \dot{\rho}_r & \dot{\rho}_p \\
\dot{\gamma}_p & \dot{\gamma}_p & \dot{\gamma}_r & \dot{\gamma}_p \\
\dot{\phi}_p & \dot{\phi}_p & \dot{\phi}_r & \dot{\phi}_p
\end{bmatrix} +
\begin{bmatrix}
y_\xi & y_\xi & y_\xi \\
\dot{\rho}_\xi & \dot{\rho}_\xi & \dot{\rho}_\xi \\
\dot{\gamma}_\xi & \dot{\gamma}_\xi & \dot{\gamma}_\xi \\
\dot{\phi}_\xi & \dot{\phi}_\xi & \dot{\phi}_\xi
\end{bmatrix} \xi
\]  

Equation for coupled Lateral - Directional Motions

In lateral – directional motions,
\( y_p \) = Concise side force due to sideslip, \( l_p \) = Concise rolling moment due to sideslip, \( y_\rho \) = Concise yawing moment due to sideslip, \( y_\phi \) = Concise side force due to roll rate, \( y_\phi \) = Concise side force due to roll angle, \( l_\rho \) = Concise rolling moment due to roll angle, \( n_\rho \) = Concise yawing moment due to roll rate, \( n_\rho \) = Concise yawing moment due to roll angle, \( y_\gamma \) = Concise side force due to yaw rate, \( l_\gamma \) = Concise rolling moment due to yaw rate, \( n_\gamma \) = Concise yawing moment due to yaw rate, \( v \) = body frame Y-velocity, \( p \) = roll rate, \( r \) = yaw rate, \( \gamma \) = roll angle, \( \gamma \) = Concise yawing force due to aileron, \( l_\gamma \) = Concise rolling moment due to aileron, \( n_\gamma \) = Concise yawing moment due to aileron, \( y_\phi \) = Concise rolling moment due to rudder, \( n_\phi \) = Concise yawing moment due to aileron angle perturbation, \( \xi \) = Aileron angle perturbation.

A Solution of the Equation for the Combined Lateral-Directional Dynamics Was Summarized as

\[
y(t) = I_x(t)
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\nu \\
\rho \\
\gamma \\
\phi
\end{bmatrix}
\]

Equation 4 A solution of the equation for the coupled Lateral-Directional dynamics

3. Modelling Notations

The inputs to the model of the light aircraft described by its motions or dynamics were the control surface deflections on the elevator, rudder and aileron. These inputs made changes on the properties of the aircraft. Therefore, the ABT-18 light aircraft was modelled such that it was described by its states. The 12 states described in the research project were given below as:

- \( X \): inertial X coordinate (meters north)
- \( Y \): inertial Y coordinate (meters east)
- \( H \): altitude in inertial coordinates (meters)
- \( U \): body frame X – velocity (meters/second)
- \( V \): body frame Y – velocity (meters/second)
- \( W \): body frame Z – velocity (meters/second)
- \( \phi \): roll angle (radians)
- \( \theta \): pitch angle (radians)
- \( \psi \): heading angle (radians)
- \( P \): roll rate (radians/second)
- \( Q \): yaw rate (radians/second)
- \( R \): pitch rate (radians/second)
- \( \dot{\phi} \): roll rate (radians/second)
- \( \dot{\theta} \): pitch rate (radians/second)

As such, the 12 states described in the research project were:

1. \( x_1 = X \)
2. \( x_2 = Y \)
3. \( x_3 = H \)
4. \( x_4 = U \)
5. \( x_5 = V \)
6. \( x_6 = W \)
7. \( x_7 = \phi \)
8. \( x_8 = \theta \)
9. \( x_9 = \psi \)
10. \( x_{10} = P \)
11. \( x_{11} = Q \)
12. \( x_{12} = R \)

Table 2. Characteristics of the PID gains

<table>
<thead>
<tr>
<th>CI RESPONSE</th>
<th>RISE TIME</th>
<th>OVERSHT</th>
<th>SETTLING TIME</th>
<th>S-S ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_p )</td>
<td>Decrease</td>
<td>Increase</td>
<td>Small Change</td>
<td>Decrease</td>
</tr>
<tr>
<td>( K_i )</td>
<td>Decrease</td>
<td>Increase</td>
<td>Eliminate</td>
<td>Decrease</td>
</tr>
<tr>
<td>( K_d )</td>
<td>Small Change</td>
<td>Decrease</td>
<td>No Change</td>
<td></td>
</tr>
</tbody>
</table>

It was observed that the correlations may not be exactly accurate, because \( K_p \), \( K_i \), and \( K_d \) are dependent on each other. In fact, changing one of these variables can change the effects of the other two. For this reason, the table should only be used as a reference when you are determining the values for \( K_i \), \( K_p \), and \( K_d \). [11]

Since, the project focused on the integration of control and autopilot system into the Air Beetle ABT-18 aircraft; a small
trainer aircraft, proposed to be converted to an UAV, the control was conducted in the ground control station (GCS) and the autopilot design was embedded in the FMS of the aircraft set in the Ground Control System. The autopilot design integrated was therefore, simple but covered all the necessary aspects needed to fly an aircraft with lesser flying instruments than those made by world class companies with modern instruments. The use of computer software to control the ABT-18 UAV in the GCS was applied. The software was able to read the aircraft's current position and then control the FCS, which is also the autopilot system, to guide the aircraft through a defined trajectory or way points as can be seen in Figure 2.

3.3. Designs and Simulations

In the design of the autopilot system for the control of the aircraft, the aircraft dynamic characteristics such as its equations of motion were needed. To come up with these equations, some steps were undertaken, these are:

i. Using AVL to attain the model of the ABT-18 Aircraft or UAV.

ii. Using AVL to investigate the aircraft stability characteristics on the longitudinal and lateral directional axis.

iii. Deriving the aircraft’s longitudinal and Lateral directional dimensionless derivatives using AVL.

iv. Converting these dimensionless derivatives to concise equations.

v. Inserting the concise derivatives into a state space equation.

vi. Augmenting the state space equations to achieve some states such as the altitude, sideslip and heading angle.

vii. Transforming the state space equation from that of radians output to degree output for the longitudinal and lateral directional axes.

viii. Using the transformed state space equations to derive transfer functions for the longitudinal and lateral directional axes.

ix. Importing these derived transfer functions into the Matlab/Simulink environment to simulate and analyse the performance of the aircraft Simulink model.

x. Controlling the aircraft response to input controls using successive loops and controllers in the Simulink environment.

xi. Simulation of the longitudinal and lateral directional autopilot control by assigning targets to the autopilot.

3.3.1. Athena Vortex Lattice Modelling

The specifications for wings, rudder, ailerons and elevators as drawn by CAD was imported to create a model of the lifting surfaces of the aircraft. The dimensions used included the mass of the aircraft, wing dimensions and tail dimensions. These were used because they are the major contributors to the aircraft control. The Figure 3 represents the aircraft’s ABT-18 UAV in the AVL as used for the computational analysis.
Further representation of the aircraft model showing the forces acting normal to aircraft’s lifting surface in terms of geometrical plot was also presented using AVL software. The Figure 4 represents the geometry plot of the aircraft using AVL.

In the process of investigation, the essential stability characteristic of the aircraft which is the pitching moment curve to determine the equilibrium angle of attack at the cruise speed was investigated.

### 3.3.2. Pitch Moment Curve
The pitching moment curve depicts the effect of the variation of angle of attack on the pitching moment coefficient of the aircraft. This is the characteristic of the aircraft that determines if the aircraft is longitudinally statically stable or unstable. Figure 5 shows that the gradient of the pitching moment curve is negative ($C_{ma} = -0.01936$), hence the aircraft is statically longitudinally stable. It also reveals the equilibrium angle of attack of the aircraft which is the angle at which the pitching moment coefficient is zero. Here, the equilibrium of the aircraft was gotten at about $1.6^0$ angle of attack.

### 3.3.3. Yawing Moment Curve
For the yawing moment curve, in the investigation of the lateral directional stability of the aircraft, the yawing moment coefficient was compared with the sideslip angle such that at a negative side slip, the yawing moment will be negative so as to create a restoring force to push it back to lateral equilibrium of zero sideslip and vice versa in the case of a positive sideslip angle. Figure 6 describes the stability of the ABT-18 UAV using AVL software to compute the values for yawing moment coefficient with varying sideslip angle and was seen as positive ($C_{nb} = 0.002992$).

### 3.3.4. Rolling Moment Curve
For the rolling moment curve, the stability of the aircraft around the longitudinal axis about the roll axis, the coefficient due to roll is plotted with respect to change in sideslip angle of the aircraft. It is required to be negative such that a restoring moment will be provided to return the aircraft to its equilibrium roll attitude after disturbance occurs on this axis. Figure 7 depicts the roll moment curve of the ABT-18 UAV showing a negative curve ($C_{lb} = -0.00129$).

### 4. The ABT-18 UAV Aircraft Dynamics
The concise derivatives of the ABT-18 aircraft longitudinal dynamics were obtained from the aerodynamics coefficients by the AVL Software. The AVL produces the dimensionless derivatives for both longitudinal and coupled lateral directional axes. These dimensionless derivatives were
converted to concise derivatives by a set of equations. The concise derivatives for the longitudinal axis of the aircraft were obtained as:

\[
\begin{align*}
    x_u &= -0.00291, \
    m_u &= 0.7538, \
    x_w &= 0.2644, \
    m_w &= -3.2634, \
    x_d &= -0.8119, \
    m_d &= -1811.491, \
    x_i &= 2.604, \
    m_i &= -0.0848, \
    x_q &= 4.7771, \
    m_q &= 0.996, \quad z_d = 0.2677
\end{align*}
\]

While the concise derivatives for the coupled lateral directional axis of the aircraft were given as

\[
\begin{align*}
    y_v &= -0.1062, \
    n_v &= 0.0, \
    y_p &= 1.3286, \
    n_p &= 0.00158, \
    y_t &= 1.801449, \
    n_t &= 0.996, \quad \theta_v = 0.2677, \
    \phi_i &= 0.00158, \
    \theta_i &= -50.8927, \
    \phi = -0.1104, \
    \theta &= 9.8100, \
    \phi_t &= 51.56, \
    \theta_t &= -2366.83, \
    \phi_i &= -2.7E-11, \
    \theta_i &= -1.43E-10, \
    \phi &= 427.7244, \quad \theta = 9.88E-8, \quad \psi = 0.0, \quad \gamma = -291.107, \quad \delta = 180.1449, \quad \eta = -1458.11
\end{align*}
\]

These concise values for longitudinal dynamics were inserted into a state space equation of the aircraft with the equation below:

\[
x' = Ax + Bu
\]

Equation 6 State space equations

Therefore, the ABT-18 aircraft longitudinal state space equation was expressed as;

\[
\begin{bmatrix}
    u' \\
    w' \\
    q' \\
    \theta'
\end{bmatrix} = 
\begin{bmatrix}
    -0.00291 & 0.2644 & -0.8119 & -9.807 & 2.603 \\
    -0.1716 & -0.386 & 4.777 & 0.2677 & -2.0123 \\
    0.7538 & -3.263 & -1811.5 & -0.0848 & 0.996 \\
    0 & 0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
    u \\
    w \\
    q \\
    \theta 
\end{bmatrix} + \begin{bmatrix}
    1 \\
    0 \\
    1 \\
    0
\end{bmatrix} \begin{bmatrix}
    \eta \\
    \\phi \\
\end{bmatrix}
\]

Equation 7 ABT-18 Aircraft longitudinal state space equation

Where \( \eta \) is taking to be the elevator deflection of the aircraft. The transfer function for the longitudinal dynamics was computed by MATLAB in radians but the analysis of the autopilot angle deflections was conducted in degrees so the radians had to be converted to degrees. Hence, the transfer function became:

\[
\begin{align*}
    Ax(t) + Bu(t) & = Cx(t) + Du(t) \\
    & = \begin{bmatrix}
    \eta \\
    \phi \\
    \\theta \\
    \\psi \\
    \gamma
\end{bmatrix} \begin{bmatrix}
    1 \\
    0 \\
    0 \\
    1 \\
    1
\end{bmatrix} \begin{bmatrix}
    \phi \\
    \theta \\
    \psi \\
    \gamma \\
\end{bmatrix}
\end{align*}
\]

Equation of Altitude augmentation of longitudinal equation of motion

\( V_0 \) was the cruise speed. Thus, the state space equation changed to a 5 state matrix with the altitude (h) being added to the list of equations. The equation state matrix, therefore, became:

\[
\begin{bmatrix}
    u' \\
    w' \\
    q' \\
    \theta' \\
    h'
\end{bmatrix} = 
\begin{bmatrix}
    1 & 0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0 \\
\end{bmatrix} \begin{bmatrix}
    u \\
    w \\
    q \\
    \theta \\
    h
\end{bmatrix} + \begin{bmatrix}
    2.603 \\
    -2.0123 \\
    0.996 \\
    51.4 \\
\end{bmatrix} \begin{bmatrix}
    \eta \\
    \phi_i \\
    \theta_i \\
    \phi \\
    \theta
\end{bmatrix}
\]

Equation 13 State space equations for altitude

Equations of ABT-18 aircraft augmented longitudinal equation of motion

Thus, the dynamic response of the aircraft to a longitudinal disturbance was simulated using an impulse deflection on the elevator. It was seen that an aircraft disturbance will return to its equilibrium position after a disturbance on the longitudinal axis.
The response of the aircraft to 1 degree elevator deflection was given as shown in Figure 9.

From the response of the aircraft to 1° elevator deflection, it was observed that the steady state response which is the final value of the system after the transient response had decayed as interpreted from the graph above is given as:

\[
\begin{bmatrix}
    u \\
    w \\
    q \\
    \theta_{steady~state}
\end{bmatrix}
= \begin{bmatrix}
    -0.14 \text{ m/s} \\
    -0.023 \text{ m/s} \\
    0 \\
    0.159^\circ
\end{bmatrix}
\] (15)
ABT-18 aircraft lateral directional state space equation at steady state value

Next, for the lateral directional axis of the aircraft dynamics, we inserted the concise lateral directional coefficients into the state space equation of the aircraft with the general space equation below:

\[ x' = Ax(t) + Bu(t) \]
\[ y(t) = Cx(t) + Du(t) \]  

\[ x' = \begin{bmatrix} p' \\ r' \\ \phi' \end{bmatrix} = \begin{bmatrix} -0.1062 & -1.3286 & 50.8927 & 9.8100 \\ 161.83 & -2366.82 & 427.724 & 0.0 \\ -291.107 & 180.1449 & -1458.11 & 0.0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ y(t) = \begin{bmatrix} \beta' \\ v' \\ p' \\ r' \\ \phi' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ v \\ p \\ r \\ \phi \end{bmatrix} \]

ABT-18 lateral directional state space equation

Where \( \xi \) and \( \zeta \) are the aileron and rudder deflections of the aircraft respectively, but since the lateral and directional dynamics are coupled, the heading of the aircraft must be integrated into the equation so as to augment it and determine the stability on the directional axis. Hence, the heading angle equation of the aircraft is a function of the rate of yaw of the aircraft while the sideslip is gotten from the equation 17. For the sideslip angle,

\[ \beta = \frac{v}{\nu_0} \]

Inserting this state space equation into the MATLAB environment, the response of the states to a disturbance which is simulated to be an impulse disturbance on the aircraft was given as:

**Figure 10. Aircraft response to impulse inputs to aileron and rudder.**
It was confirmed that the disturbance on the lateral and directional axis as simulated by a 1 impulse deflection on either aileron or the rudder, had several effects on the aircraft. The impulse response of the aileron due to impulse input was seen in the sideslip as it deflected a little due to the aileron and stabilized at equilibrium after about 60 seconds while the rate of roll and yaw due to aileron impulse deflection had a sharp rise and fall within the first 5 seconds. This sharp rise in the yaw and roll rates caused the roll and yaw angle to change due to a phenomenon called adverse yaw, but it returned back to equilibrium after about 120 seconds.

It was seen that a new yaw angle was achieved due to the impulse input on the aileron. It was also seen that the effect of deflection on the rudder can be mostly defined on the sideslip of the aircraft but it returned back to zero sideslip after about 1 second. These showed that the aircraft is lateral/directionally stable.

Thus, having determined that the aircraft was stable, the transfer function of the aircraft dynamics on the lateral directional axis was investigated, where the MATLAB command was used to compute the transfer functions from the state space equation fed to it. The transfer function in terms of radians is therefore given as:

Transfer function from input “Aileron” to output:  
\[
\frac{-0.00337 (s+6.551e004)(s+1.081e004)(s+1.135)}{(s+2444)(s+1371)(s+10.14)(s+0.03218)}
\]  
(22)

Transfer function from input “Rudder” to output:  
\[
\frac{-0.00225 (s+1.694e005)(s+2296)(s+8.1e-005)}{(s+2444)(s+1371)(s+10.14)(s+0.03218)}
\]  
(27)

Equations of ABT-18 UAV Lateral directional transfer functions in radians

The generated transfer functions were then multiplied by 0.0175 so as to convert it from the radian output to degree output. Hence, the transfer function in terms of degree was given as:

Transfer function from input “Aileron” to output:

- **Sideslip:**  
  \[
  \frac{-5.8975e-005 (s+6.551e004)(s+1.081e004)(s+1.135)}{(s+2444)(s+1371)(s+10.14)(s+0.03218)}
  \]  
  (32)

- **Roll rate:**  
  \[
  \frac{3.269 (s+1468)(s+10.46)}{(s+2444)(s+1371)(s+10.14)(s+0.03218)}
  \]  
  (33)

- **Yaw Rate:**  
  \[
  \frac{0.15316 (s+6210)(s^2+1.509s+10.07)}{(s+2444)(s+1371)(s+10.14)(s+0.03218)}
  \]  
  (34)

- **Roll angle:**  
  \[
  \frac{-3.269 (s+1468)(s+10.46)}{(s+2444)(s+1371)(s+10.14)(s+0.03218)}
  \]  
  (35)

- **Yaw angle:**  
  \[
  \frac{-0.15316 (s+6210)(s^2+1.509s+10.07)}{(s+2444)(s+1371)(s+10.14)(s+0.03218)}
  \]  
  (36)

Transfer function from input “Rudder” to output:

- **Lateral Velocity:**  
  \[
  \frac{2.9975e-005 (s+1.694e005)(s+2296)(s+8.1e-005)}{(s+2444)(s+1371)(s+10.14)(s+0.03218)}
  \]  
  (37)

- **Roll rate:**  
  \[
  \frac{-0.037905 (s+117)(s+113.8)}{(s+2444)(s+1371)(s+10.14)(s+0.03218)}
  \]  
  (38)

- **Yaw Rate:**  
  \[
  \frac{0.12892 (s+2314)(s^2+0.06274s+0.3232)}{(s+2444)(s+1371)(s+10.14)(s+0.03218)}
  \]  
  (39)

- **Roll angle:**  
  \[
  \frac{-0.037905 (s+117)(s+113.8)}{(s+2444)(s+1371)(s+10.14)(s+0.03218)}
  \]  
  (40)

- **Yaw angle:**  
  \[
  \frac{-0.12892 (s+2314)(s^2+0.06274s+0.3232)}{(s+2444)(s+1371)(s+10.14)(s+0.03218)}
  \]  
  (41)

Equations of Lateral directional transfer function in degrees

The response of the aircraft with respect to a 1 degree input of the aileron and rudder was also determined. Hence the response and characteristics of the aircraft was given as in Figure 11:

The Figure 11 depicted the responses of the rudder and the aileron step input but it was inferred that the effect of the rudder deflections with respect to the aileron deflections were small. Hence, analysing the responses from aileron step deflections, we saw that the sideslip attained a steady state after about 100 seconds while the roll rate peaked before it returned to zero rate of roll, however, the yaw rate continued to rise till it arrived at its steady state after about 120 seconds.

Also, the roll angle rose till it arrived at its steady roll angle state at about 100 seconds which was unlike the yaw angle as it continued to change its heading as far as the roll angle was held.
4.1. ABT-18 UAV Autopilot Design

Having gotten the equations of motion of the aircraft, the transfer functions and investigating the responses of the aircraft to different control surface deflections, the design of the control for the autopilot was done using the successive loop theory such that the autopilot will have an inner loop feedback control to control certain aircraft states such as roll rate, pitch rate etc. while the outer loop was designed to control the inner loop so as to achieve its desired target.

4.2. ABT-18 UAV Longitudinal Autopilot

In the design of this control, a look into the pitch attitude hold and the altitude hold of the aircraft was considered where the block diagram of the longitudinal autopilot was given as in Figure 12.

The longitudinal autopilot was designed to take input target from the flight management system (FMS) and this target was set as reference for the autopilot and also interfaces with the flight control system as shown above in the Figure 12. The deflection commands to the elevator were also received from the FMS as it overrides the operation of the autopilot. The signals generated from the output of the aircraft dynamics were fed back to the autopilot through sensing systems or devices such as rate gyro for pitch rate to control the rate of pitch of the aircraft, Attitude and Heading Reference System to control the pitch attitude while the altimeter feeds back the altitude status for altitude control. The longitudinal autopilot was designed for ABT-18 UAV to control altitude and pitch attitude of the aircraft. The design
details of the autopilot are as shown in the next designated phases of the project;

4.2.1. Pitch Attitude Hold

Since, the pitch attitude hold mode prevents pilots from constantly having to control the pitch attitude, especially, in turbulent air, this can get tiring for the pilot. This system uses the data from the Attitude and heading reference system (AHRS) as feedback and controls the aircraft through the elevators to achieve the desired target. The Figure 13 of the Pitch attitude hold using MATLAB/Simulink model with PID controllers and the graph is given as Figure 14 respectively:

The pitch attitude of the aircraft as seen from the equations of motion was solely dependent on the pitch rate of the aircraft, hence, the pitch attitude controller was a loop around the pitch rate of the aircraft. An integrator was used on the pitch rate so as to set the steady state response of the pitch rate to be at zero without any steady state error but since the PID controllers of the pitch attitude’s second phase loop would provide an integrator, placing the integrator pole at zero made the system marginally stable, therefore, the pole was shifted to have a position at -0.001.

The pitching attitude of the aircraft was controlled with the use of a PID controller to tune the response of the pitch attitude with respect to the degree of deflection of the elevator. The Figure 14 therefore, shows the rise time, settling time, and other response characteristics of the controller. It was observed that the P-controller speeds up the system but increases overshoot, I-controller has little effect on overshoot but drives the steady state error to zero while the D-controller reduces the overshoot but increases the settling time. Hence, due to the controller effects, the Rise time is 3.3s, Settling time is 42.5s and Overshoot is 7.48%.
4.2.2. Altitude Hold

The altitude hold mode prevents pilots from constantly having to maintain their altitude. The feedback was received from the altimeter and the system then uses the elevator to control the altitude to the target as specified to the autopilot system. The Figure 15 shows the altitude hold using MATLAB/Simulink model and its graphical response.

From the equation of motion earlier generated, it was deducted that the altitude of the aircraft was dependent on the pitch altitude, hence, the altitude loop of the autopilot was successive on the pitch attitude of the aircraft.

The controller used in the longitudinal altitude control of the aircraft was also a PID controller with its characteristic value given for Rise time as 13.3s, settling time as 60.1s and Overshoot as 4.6%. Hence, the auto controls of the longitudinal autopilot for the scope of the research was limited to the altitude and pitch attitude hold modes respectively, however, this can be improved by adding the airspeed hold mode.

4.3. ABT-18 UAV Lateral Directional Autopilot Design

The Figure 17 illustrates the model diagram of the lateral directional autopilot such that the successive loop was over the aileron controls; the roll mode, roll angle and heading control mode. The loop around the rudder controls the sideslip of the aircraft.
4.3.1. The Roll Angle Hold Mode

Here, the roll angle hold mode prevented the pilot from constantly having to adjust or control the roll angle during a turn. It used the roll angle gyroscope as sensor which was fed into the Air Data computer before being sent to the autopilot system. Hence, to control the roll angle, the roll rate must first be controlled using the PID controller as shown in Figure 18.

Figure 17. Lateral directional autopilot control block diagram.

Figure 18. Roll rate controller design for lateral directional autopilot.

Figure 19. Tuned response using PID controllers for Roll rate controller.
The loop assisted in improving the response of the roll rate to autopilot control so as to achieve a smooth operation of the autopilot design. The roll angle or attitude of the aircraft was seen from the equations of motion as solely dependent on the roll rate of the aircraft, hence, the roll attitude controller as seen in the Figure 20 is a loop around the roll rate of the aircraft.

![Figure 20. Roll attitude controller.](image)

The roll angle hold attitude of the aircraft is controlled with the use of a PID controller to tune the response of the roll attitude with respect to the degree of deflection of the aileron. The Figure 21 shows the Rise time of 12.9s, Settling time of 38.2s, Overshoot of 8.35% and other response characteristics of the controller.

![Figure 21. Response of roll angle hold autopilot design with respect to PID controller.](image)

4.3.2. Coordinated Roll Angle Hold Mode

The coordinated roll angle hold mode is an extension of the roll angle hold mode. This usually results in a coordinated turn, thus giving the aircraft less drag and the passengers more comfort for conventional or manned aircraft. The coordinated roll angle hold mode of the autopilot uses the sideslip sensor as input into the Air Data Computer and sends a signal to the rudder. This was also applicable to the ABT-18 UAV autopilot design as one of its modes and PID controller was used to determine the model and response as shown in Figure 22 and Figure 23 respectively.
In the case of roll attitude of the aircraft, the possibility of the aircraft to slip was high, hence the autopilot was designed to set the sideslip of the aircraft to zero. This was done by controlling the sideslip of the aircraft to have a zero steady state by sending a loop around the rudder using the AHRS as the feedback system. The controller sets the sideslip to a constant zero during turn or bank of the aircraft. This controller was tuned to a Rise time of 0.8s, Settling time of 1.45s and Overshoot as 7.88%.

4.3.3. Heading Hold Mode
The heading hold angle control mode controls the heading. It sends a signal to the (coordinated) roll angle hold mode, telling it which roll angle the aircraft should have. This roll angle is maintained until the desired heading is achieved. To achieve the heading hold, the Attitude and Heading Reference System was used as a feedback system and to correct the aircraft to the reference heading input.
From the equation of motion earlier defined, it was deduced that the heading angle of the aircraft was dependent on the roll attitude of the aircraft, hence, the heading loop of the autopilot was successive on the roll attitude of the aircraft.

The controller used in the heading control of the aircraft was therefore, a PID controller with its characteristic value given here, with a Rise time of 1.2s, settling time of 2.5s and Overshoot of 5.6%.

### 5. Guidance Mode Integrating Longitudinal and Lateral Controls

The guidance mode of the aircraft describes the aircraft integration to the input systems of the autopilot design. The designed autopilot receive the desired heading, desired altitude and optionally roll angle and pitch angle of the aircraft, thus, giving mode select switches that would be used to engage or disengage modes of the aircraft if not indicated by the FMS of the aircraft.

#### 5.1. Simulation Results

The simulation of the autopilot in MATLAB/Simulink was done with the following results obtained for both the longitudinal and lateral controls respectively.
Figure 26. Complete System view of the longitudinal and lateral autopilot.

Figure 27. Longitudinal simulation of autopilot.
The Figure 27 above is a simulation on the longitudinal autopilot where an altitude of 40m was fed to the autopilot system; the aircraft response is such that the pitch angle increases until the aircraft attains the desired altitude after about 40 seconds and holds that altitude as the aircraft pitches back down. This altitude was also changed to 100 meters and the autopilot controlled the aircraft till 100 meters as seen above.

The lateral directional axis simulation is as shown in the Figure 28. The aircraft sideslip was controlled to minimal as fed to the autopilot system while the aircraft changes its heading to 8 degrees as autopilot input varies; while roll angle changes to about 35 degrees to control the aircraft to desired heading. The roll angle was also simulated as a roll angle of 10 degree been fed and the resultant heading due to this input is a 2 degree heading.

The autopilot of this aircraft is therefore, designed to operate only in cruise phase of the aircraft. This design stage began from the modelling of the aircraft on the AVL software and determination of the aircraft transfer function in terms of degrees instead of the default radian mode of the AVL software.

The transfer function was then used to determine the response of the aircraft to different control surface deflections. These responses were used to determine the steady state values for different states of the aircraft due to the deflection of these control surfaces.

5.2. Conclusion

The article looked into the background of the stability characteristics of a light aircraft, the software used in the computation of the aircraft dynamic characteristics and derivatives, the equation of motion for a fixed wing aircraft such as the ABT-18 aircraft and its modified form. The functional block diagram of the autopilot system and the system architecture was obtained so as to guide the system design. AVL software was used to draw the aerodynamic model of the aircraft with emphasis laid on the lifting surfaces of the aircraft. The model was used to derive the aircraft’s aerodynamic derivatives which includes the dimensionless derivatives, the stability axis derivatives, the variation of the aerodynamic coefficients at different angles of attack and sideslips. These values of different angles of attack and sideslips were used to investigate the stability of the aircraft in the longitudinal and lateral directional axis of the aircraft. The dimensionless derivatives were used to compute the concise derivatives of the aircraft, where the state space equations computed with MATLAB, supplied the transfer function of the aircraft in degrees. The transfer functions were used to determine the response of the aircraft to changes in the control surfaces of the aircraft.

After this stage, various responses of the aircraft were shown by the different autopilot modes where controllers were designed to augment the system stability. Using PID controllers so as to attain a desired output with feedbacks fed from the feedback systems, depending on the state that is chosen, desirable longitudinal and coupled lateral directional stabilities were successfully obtained showing that the aircraft, ABT-18 UAV is dynamically and statically stable.

The successive looping of the states of the aircraft to close the system and improve the response characteristics of the aircraft by looking at some selected important modes of the autopilot in terms of longitudinal autopilot control and coupled lateral directional autopilot control was investigated. This is done with PID controllers used to improve the dynamics of the aircraft enabling the state of the aircraft to track a target as the reference point of the aircraft. Here,
engaging the FCS disengages the autopilot system. In conclusion, the ABT-18 UAV had satisfied the conditions necessary to deem it longitudinally and laterally stable.

References


